B.sc Part II (Honrs)

Cosets.

Definition:- Suppose H is a Subgroup of a group $(G, \cdot)$. Let $a \in G$ be arbitrary we define

$$aH = \{ah : h \in H\}, \; Ha = \{ha : h \in H\}.$$

Evidently $aH \subset G$, $Ha \subset G$

aH is called left Coset of H in G generated by a.
Ha is called right Coset of H in G generated by a.

If e is the identity for G, then $e \in H$ is also identity for H.

$$a = ae \in aH, \quad a = ea \in Ha.$$

This proves that any left coset or right coset of H in G is not empty.

Observe that $He = H = eH$

Hence H itself is right as well as left Coset.

If the group $(G, \cdot)$ is abelian, then $ah = ha \; \forall \; h \in H$

So that $aH = Ha \; \forall \; a \in G$.

It does not mean that we can never find subgroups H of a non-abelian group G such that $aH = Ha$ for any $a \in G$.

This is illustrated by an example.

In general $Ha \neq aH$.

Example(2):- Let $H = \{3n : n \in Z\}$ be a subgroup of commutative group $(Z, +)$. Then $H = \{0, \pm 3, \pm 6, \pm 9, \ldots\}$.

$1 \in Z$, $H+1 = \{h+1 : h \in H\}$

$= \{3n+1 : n \in Z\}$

$= \{1, 4, 7, 10, 13, \ldots, -2, -5, -8, -11, \ldots\}$

for any $h \in H \Rightarrow \exists\, n \in Z$. such that $h = 3n$

$2 \in Z$, $H+2 = \{3n+2 : n \in Z\}$

$= \{2, 5, 8, 11, 14, \ldots; -1, -4, -7, -10, \ldots\}$

$3 \in Z$, $H+3 = \{3n+3 : n \in Z\} = \{3(n+1) : n \in Z\}$

Thus $H+3 = H$, $3 \in H$

Similarly

$H+4 = H+1$, $4 \in H+1$

Generalising this result, We have

$H+n = H+a$ if $n \in H+a$.

Thus $\exists$ three disjoint right cosets namely

$H, H+1, H+2$.

It is easy to verify that

$Z = H \cup (H+1) \cup (H+2)$.

Example-(2) $H = \{n+a : a \in Z\}$ is a subgroup of $(Z, +)$, Where $n \in Z$ is fixed. Then We can show that $\exists$ n distinct right Cosets, namely

$H, H+1, H+2, \ldots, H+n-1$

and $Z = H \cup (H+1) \cup (H+2) \cup \ldots \cup (H+n-1)$.

Anjani Kumar Singh
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